use of mean cooling rates of up to 30 deg/sec [6]. The film samples were melted between glass covers, with temperature measurement by means of microthermocouples placed directly in the sample. In the present work, the mean cooling rate is interpreted as the technological parameter v in comparing the data of [6] with the present calculation scheme.

NOTATION

α, degree of crystalline structure; t, time; T, temperature; K(t), temperature dependence of overall rate of crystallization; $\alpha_e(T)$, temperature dependence of equilibrium degree of crystalline structure; f [α, $\alpha_e(T)$], formal law characterizing the rate of the process as a function of the degree of crystal structure; k_0, preexponential factor; c_0, parameter characterizing the type (geometry) of growing structure; ψ, U, effective energetic constants characterizing nucleation and growth, respectively; T_m, equilibrium melting point; T_c, vitrification, temperature; α_∞, final degree of crystallization structure; v, rate of temperature variation.

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NOISE SHEAR WAVES IN NONLINEAR SMECTIC LIQUID CRYSTAL

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The propagation of a narrow-band shear wave in a smectic liquid crystal is studied. It is shown that the nonlinearity makes possible a random phase modulation. The temporal correlation function, the characteristic correlation time, and the width of the spectrum are calculated. The relation between the width of the spectrum, the intensity of the wave, and the distance traversed is determined. This makes it possible to determine the coefficient of cubic nonlinearity of a smectic liquid crystal (SLC) by measuring the relative increase of the width of the spectrum at an arbitrary point.

In [1-3] it is shown that it is most important to take into account nonlinear factors in the behavior of liquid crystals when studying their smectic phase.

The propagation of regular shear waves in a nonlinear smectic was studied in [4]. Some mechanisms of the propagation of noise shear waves in SLC were analyzed in [4].

In order to describe the dynamics of SLC (smectic A) it is convenient to introduce the smectic variable $W(r, t) = (z + u)/l$. Here l characterizes at equilibrium $W_0 = z/l$ a system of layers perpendicular to the z-axis.

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Two types of sound waves can propagate in the SLC — longitudinal waves, determined by the compressibility, and shear waves (second sound), which are associated with the displacement of the layers and correspond to the condition of incompressibility [3].

Propagation of a shear plane wave in a nonlinear smectic, under the assumption that the smectic is incompressible, is described by the equation [4]

$$\frac{4\rho K_0}{\eta^2}\frac{\partial^2\Psi}{\partial\tau^2}+\frac{\partial^4\Psi}{\partial\xi^4}=\frac{3\alpha}{2}\left(\frac{\partial\Psi}{\partial\xi}\right)^2\frac{\partial^2\Psi}{\partial\xi^2}. \tag{1}$$

The equation (1) is written in the dimensionless variables

$$u=\lambda\Psi,\ \xi=(\pi/\lambda d)^{1/2}x,\ \tau=(2\pi K_0/\eta\lambda d)\,t.$$

In the linear approximation the solution of Eq. (1) can be represented in the form of a collection of harmonics, whose frequencies (ω) and wave numbers (k) are related by the dispersion relation

$$D(\omega,\ k)=\frac{4\rho K_0}{\eta^2}\omega^2-k^4=0. \tag{2}$$

Shear waves exhibit strong dispersion, since their phase velocity $v_{ph} = \omega/k \neq$ const and different harmonics propagate with different velocities. For this reason, in the case of weak nonlinearity the solution of Eq. (1) is close to the solution of the linear problem and it can be represented as a collection of quasi-harmonics [5, 6]. In addition, for systems with cubic nonlinearity the effect of self-action usually predominates over generation of higher-order harmonics and the latter can be neglected [5]. This makes it possible to seek the solution of Eq. (1) in the form of a single harmonic with complex amplitude (A) varying slowly in space and time:

$$\Psi=A(\varepsilon\xi,\ \varepsilon\tau)\,e^{i(\omega\tau-k\xi)}+\text{c.c.} \tag{3}$$

Here $(\partial A/\partial\xi)/kA \sim (\partial A/\partial\tau)/\omega A \sim \varepsilon \ll 1$.

Using the method of averaging [6], we can transfer from Eq. (1) to a truncated equation for the complex amplitude:

$$\frac{\partial A}{\partial\xi}+\frac{1}{v_{gr}}\frac{\partial A}{\partial\tau}=-i\beta|A|^2A, \tag{4}$$

where $v_{gr} = d\omega/dk$ is the group velocity, $\beta = -3\alpha k/4$, and the boundary condition

$$A(0,\ \tau)=A_0(\tau) \tag{5}$$

must be added.

Let $A_0(\tau)$ be a narrow-band random process with relative spectrum width $\Delta\omega/\omega_0 \ll 1$. The solution of the problem (4) and (5) can then be written in the form

$$A(\xi,\ \tau)=A_0(\tau-\xi/v_{rp})\exp\{-i\beta|A_0(\tau-\xi/v_{rp})|^2\xi\}. \tag{6}$$

The correlation function for the solution (6) at an arbitrary point is given by the expression

$$\begin{aligned}B(\theta,\ \xi)&=\langle A(\tau,\ \xi)A^*(\tau+\theta,\ \xi)\rangle=\\&=\langle a_0a_{0_\theta}\exp\{i[\beta\xi(a_{0_\theta}^2-a_0^2)+\varphi_0-\varphi_{0_\theta}]\}\rangle,\end{aligned} \tag{7}$$

where

$$A=a\exp(i\varphi),\ a_0=a_0(\tau-\xi/v_{gr}),\ a_{0_\theta}=a_0(\tau+\theta-\xi/v_{gr}).$$

In order to calculate the correlation function (7) it is necessary to know the four-dimensional distribution function

$$\Phi(a_0, a_{0_\theta}, \varphi_0, \varphi_{0_\theta}) =$$
$$= \frac{a_0 a_{0_\theta}}{\pi^2 I^4 [1 - b_0^2(\theta)]} \exp\left\{-\left[\frac{a_0^2 + a_{0_\theta}^2 - a_0 a_{0_\theta} b_0(\theta)\cos(\varphi_0 - \varphi_{0_\theta})}{I^2(1 - b_0^2(\theta))}\right]\right\}. \tag{8}$$

Here $I = \langle {}_0{}^2 \rangle^{1/2}$ is the standard deviation; $b_0(\theta) = \langle a_0 a_{0_\theta} \rangle / I^2$ is the normalized correlation function of the random process at the boundary of the medium. For definiteness we shall assume that it is Gaussian:

$$b_0(\theta) = \exp\{-4(\ln 2)(\theta/\theta_0)^2\}. \tag{9}$$

From Eqs. (7) and (8) we determine the normalized correlation function for a random process in the medium [7]:

$$b(\theta, \xi) = \frac{B(\theta, \xi)}{B(0, \xi)} = \frac{b_0(\theta)}{[1 + (\beta I^2 \xi)^2 (1 - b_0^2(\theta))^2]^2}. \tag{10}$$

The characteristic correlation time θ_c of the wave can be found from the condition that the correlation function has decreased to one half its initial value:

$$b(\theta_c, \xi) = \frac{b(0, \xi)}{2} = 1/2. \tag{11}$$

If all characteristics of the smectic are known, then the expression (10) makes it possible to determine, using Eqs. (9) in (11), how the width of the spectrum $\Delta\omega_c$ of the noise shear wave changes as the wave propagates in the nonlinear medium, since $\Delta\omega_c/\Delta\omega_0 \sim \theta_c/\theta_0$, where $\Delta\omega_0$ is the width of the spectrum of the random signal at the boundary.

The nonlinearity coefficient α is given by the formula

$$\alpha = \left\{\frac{[2b_0(\theta_c)]^{1/2} - 1}{[1 - b_0^2(\theta_c)]^2 I^4 \xi^2}\right\}^{1/2}, \tag{12}$$

i.e., the expression (10) also makes it possible to solve the inverse problem — to calculate the coefficient of cubic nonlinearity of the smectic from the change in the width of the spectrum of a noise shear wave.

NOTATION

Here W is the smectic variable; l is the equilibrium distance between the smectic layers; u is the displacement of the layers along the z-axis; z is the coordinate; t is the time, Ψ, ξ, and τ are dimensionless variables; η is the coefficient of viscosity; ρ is the density; d is the thickness of the smectic; α is the coefficient of nonlinearity; K_0 and B_0 are the shear moduli; $\lambda = (K_0/B_0)^{1/2} \sim 1$; ω is the frequency; k is the wave number; v_{ph} is the phase velocity; v_{gr} is the group velocity; A is the complex amplitude of the quasi-harmonic wave; a and φ are the real amplitudes and phase; $\Delta\omega/\omega_0$ is the relative width of the spectrum; B is the correlation function; Φ is the distribution function; I is the standard deviation; b_0 is the normalized correlation function; and θ_c is the characteristic correlation time.

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